SOUNDFIELD SYNTHESIS FOR GENERAL ENCLOSURES

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ABSTRACT

While it may be many years before high resolution active acoustic boundaries will be widely available, accurate soundfield control presents an interesting theoretical problem. We investigate and compare several different methods, including existing methods and two new ones, that aim to find the driving functions for sources on a general boundary, such that any given soundfield is reproduced as accurately as possible everywhere within. The methods considered include High Order Ambisonics, Wavefields, boundary element modeling, a modal boundary decomposition approach, and pressure control points. Finally a method is presented, referred to here as Distributed Modal Constraints, in which multiple regions are constrained by modal expansions simultaneously.

1. INTRODUCTION

To provide the background to later discussion we first provide an overview of the properties of the most widespread existing soundfield reproduction systems.

1.1. Ambisonics

High Order Ambisonics is based on the Fourier-Bessel expansion (FBE) of a soundfield about a point, either in two or three dimensions [1, 2]. The effectiveness of the expansion is greatly increased by the radial properties of the basis functions. When these have order greater then N, they are highly suppressed for radius $r < N/k$ in either 2 or 3 dimensions, where k is the wavenumber. This means that a given spherical volume is very accurately described by a finite number of basis functions, unless the lower orders are relatively suppressed, as happens close to a singularity.

In the decoding process the sum of the FBEs of the contributing loudspeaker sources is matched to the desired FBE, a process generally known as mode-matching. Provided there are enough loudspeakers the accuracy of the reproduced FBE can be controlled accurately, with small interference from unwanted higher orders. Usually the speakers are arranged in a circular or spherical pattern, which simplifies and optimizes the decoding process. If the speaker boundary is non-spherical, it would be desirable to reproduce a soundfield over all parts of the interior. This is not possible however, because the FBE expansion of each source is valid only as far as the radius extending from the FBE centre to the source, due to the FBE field being sourceless. Beyond this the field takes on a form that does not match the actual field produced by the real source, and so mode matching in

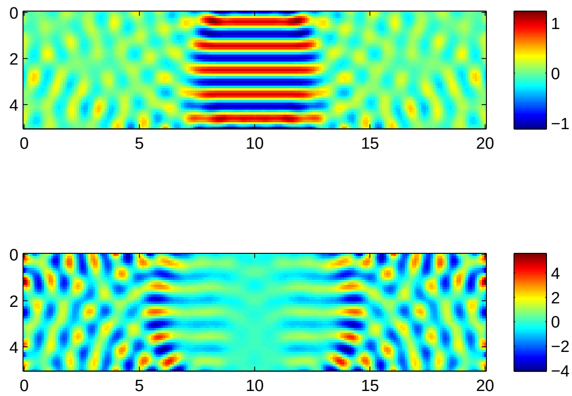

Figure 1:

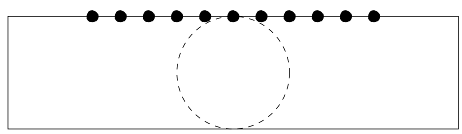

Figure 2: Limit of representation of the closest source by an FBE at the centre of a rectangular array.

this region fails. Fig. 1 shows a $(20\lambda, 5\lambda)$ dimension rectangular array with attempted order 15 and order 25 reconstruction of a 2D planewave field directed downwards, expanded about the centre. $N = 15$ reconstructs well. $N = 25$ doesn't extend the reconstructed region at all, and the existing region is disrupted. Fig 2 shows schematically the limit of representation area for the closest source.

1.2. Wavefield synthesis

The original formulation of wavefield synthesis is based on the Raleigh's integral, and expresses how a soundfield half-space can be driven by a continuum of monopole sources on the boundary, [3, 4]. To realize this practically the source region must be made finite, and sampled at a finite number of points, resulting in a finite region that is accurately reproduced. Unlike the Ambisonic system, there is no direct control over the location and shape of this region. A single linear boundary can only produce planewaves that are outward going, in other words that cover half the direction space. Ambisonics can reproduce planewaves over all directions. This implies that linear wavefield system can reproduce focused sources oriented away from the line array, but not point sources, whereas Ambisonics can synthesize both in any orientation, [5, 6]. Linear wavefield systems can be extended by combining several line arrays to make an enclosure.

2. SIMPLE SOURCE

The simple source formulation constructs a monopole source continuum on any closed boundary to reproduce any desired freefield, [7]. It can be seen as a generalization of the Rayleigh Integral. The construction involves finding a source located in the boundary whose field matches the freefield on the boundary. This can be viewed as a scattering problem [8], which also shows that the solution always exists. In practice the driving function must be sampled, as for the wavefield. Because of this the accuracy of the soundfield is not controlled directly. In the limit of closely spaced sources, both Ambisonics and wavefield reproduction can be seen as examples of the simple source formulation.

2.1. Modal boundary decomposition

Calculation of the source field using scattering could be achieved in principle by a finite element boundary solver. The fast multipole method has led recently to the availability of fast solvers such as FASTBEM, [9]. An attempt was made to avoid using such a solver to produce the source field directly, as follows. The source is written in terms of multipole source modes about an interior point. Each harmonic defines a function on the boundary. If a solution exists, as the scattering viewpoint implies it should, the boundary pressure can be expressed as a linear combination of these modal boundary functions. The boundary functions are not necessarily independent or orthogonal. The approach seems promising, but the results are disappointing. A flaw in the argument was later found. A distributed source in the interior can produce a different field on parts of the boundary compared with a source located at an interior point that has the same far-field behaviour. It could be that a variation on this approach would be more useful. However work on this was diverted by the approach described below, which proved more fruitful.

3. DISTRIBUTED CONSTRAINTS

In this section we consider solving the discrete source problem directly by applying constraints at points distributed in the interior. This is analogous to sampling a real field with spaced microphones, except that the complexity of the constraints is not limited by any microphone design considerations. As with the Ambisonics, and unlike the simple source and wavefield approaches, the discrete sources are solved for directly, which gives a potentially more accurate solution.

3.1. Pressure constraints

In [8] the desired soundfield pressure is sampled at multiple points arranged in shells. At a discrete set of resonance frequencies there exist solutions at which the sampled pressures are zero, making the soundfield undetermined by the samples.

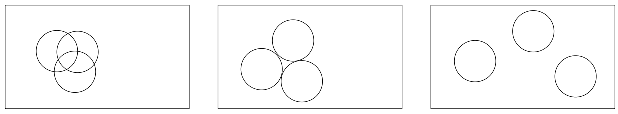

Figure 3: Modal regions, overlaped, touching and separated.

This approach was tried with various control configurations, and methods of regularization. It was found difficult to obtain even poor results for irregular boundaries. The problem seems to be that point constraints allow too much freedom, and the basic regularization methods are not then effective.

3.2. Introducing distributed modal constraints

Ambisonics is based on constraining the soundfield with a modal expansion about a single point. The question arises in the current context, what will happen if multiple modal constraints about different points are applied simultaneously? As mentioned earlier, a given order of expansion constrains a finite region very tightly, while outside the region is relatively free. This would appear to be a very efficient use of constraint variables compared with point constraints that only indirectly constrain a region. If two modal regions represent the same soundfield, then it might be possible for them to overlap and form a bigger region. This process could be extended to fill out any interior space. If modal regions are separated then we can either attempt to match them all to a single coherent field, or else each to a different field. Fig 3 illustrates the three cases, overlaped, touching and separated.

3.3. Formulation

The modal constraint of an Ambisonic decoder for freely located loudspeaker sources, in the frequency domain, is

$$c_n = \sum S_{jn}(\boldsymbol{r}_{s_j} - \boldsymbol{r}_c) s_j = \sum M_{nj} s_j \ , \tag{1}$$

where c_n the coefficients of the constrained modes and s_j are the source coefficients. Formulae here are generalized with a single modal index n that can apply to either the 2D or 3D case. $S_{jn}(\boldsymbol{r}_{s_j} - \boldsymbol{r}_c)$ is the nth component of the FBE decompostion of the jth source with location $\boldsymbol{r}_{s_j} - \boldsymbol{r}_c$ relative to the FBE centre. Often the sources are taken to be monopoles, but any function can be used, for instance to model a speaker more accurately, or incorporate the room response. The following examples all use monopole sources. The matrix M is defined for convenience. The decoder is solved by finding the pseudo inverse of M, M^+ which aims to satisfy

$$s_j = \sum M^+_{jn} c_n \ . \tag{2}$$

Regularization applied to the inversion is useful in limiting source coefficients while maintaining good accuracy. This is conveniently achieved by zeroing the singular values below a cutoff, for example using the function pinv in Matlab. The modal constraint acts like a lever for the regularization, since any significant power in unconstrained orders within a modal region results in huge power outside the region, and high source power.

To extend to constraints on multiple FBEs, we use k to index them. Then

$$c^k_n = \sum S_{jn}(\boldsymbol{r}_{s_j} - \boldsymbol{r}_{c^k}) s_j = \sum M^k_{nj} s_j \ . \tag{3}$$

Note the centre positions are indexed. If the k and n indices are combined into one index $\bar{n}$ the equation can be expressed as a single matrix equation,

$$\bar{c} = \bar{M}s \tag{4}$$

The pseudo inverse $\bar{M}^+$ of the extended matrix $\bar{M}$ attempts to satisfy

$$s = \bar{M}^+\bar{c}\ . \tag{5}$$

Again, regularization is expected to be valuable in limiting source energy. The formulation has made a shift from a centric view to a multi-centric view, and has some pleasant consequences that are now demonstrated.

3.4. Simulation examples

The above method of *Distributed Modal Constraints* (DMC), otherwise referred to informally here as *Multi-Ambisonics* or even more informally as *Splodging*, is now applied to a variety of target soundfields and boundaries. The fields resulting from the calculated source coefficients are plotted within the boundary region. The real component is plotted to show a pressure snapshot, and the absolute value is plotted to show how well the magnitude of the calculated field matches the target field. The boundaries are considered to be transparent and sources point like. For simple enclosures with interior angles at corners less than 180^o this is equivalent to having perfectly absorbing walls. In other enclosures, source fields will be diffracted at some corners, creating a more complex field over the interior. This could be included in principle in the modal decomposition of the source fields, but this is not attempted here. All the fields considered in this initial investigation are 2-dimensional, or equivalently the sources can be considered as line sources in 3-dimensions. Similar results are expected for full 3-dimensional enclosures with point sources. All planewave target examples have an amplitude of 1, and the sources are 2D monopoles defined by the Hankel function $H_0^{(1)}(r)$. The source spacing is $\lambda/2$ throughout. Larger spacing causes a rapid loss of quality in cases where resolution is needed close to the boundary, and smaller spacing causes a gradual increase in quality.

In the first example a rectangular region with dimensions $(20\lambda, 5\lambda)$ is covered by 4 modal regions whose order $N = 15 \approx 5\pi$ is just sufficient to cover the width of 5λ. The regions do not overlap but just touch. The two target fields are plane waves in different directions, both moving from right to left. The singular value cutoff is 0.1 Fig. 4 and Fig. 5 show the reproductions. The scales are marked in units of λ.

The relative error is well within 0.1 over most of the interior in both cases, including areas not covered by the modal regions. The only areas with significant error are the uncovered areas near the right edge, which is where the main energy sources are in this case. These regions could be constrained by additional small modal constraints. The total number of constraints $= 4N$ is 4 times the number of constraints for a $(5\lambda, 5\lambda)$ box. This suggests that the perimeter, in the 2D case, satisfies $P = N_T\lambda$, where N_T is the total number of modal constraints that just cover the interior.

It is striking that the reproduction has correctly interpolated across unconstrained areas without the need for overlap. When overlap is used, the gain in accuracy is small, and suggests that in general soundfields that are reasonably well behaved in a sense that is not entirely clear, can also be interpolated in the same way. Such soundfields should include focused sources, and point sources located outside the region.

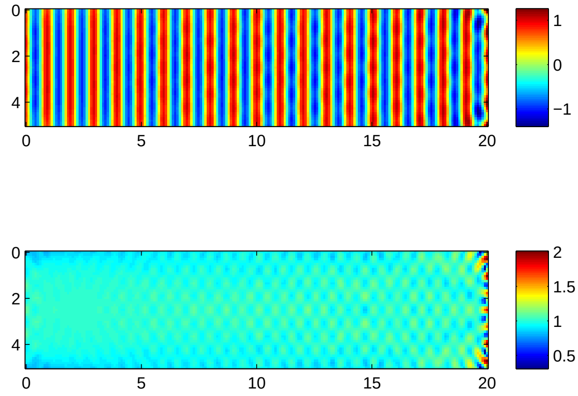

Figure 4: Reconstruction of planewave from the right, real component (top), absolute value (bottom)

The next figure, Fig. 6, shows the reproduction of an exterior point source located at $(5\lambda, 5\lambda)$ relative to the centre. Reproduction has a similar overall accuracy to that found for the plane wave. Interior sources are not presented here, as they deserve a more detailed study. The flexible constraint system opens the possibility of squeezing the solution in unusual ways. Initial findings show that focused sources can be supported without problems.

Fig. 7 shows the reproduction of a plane wave using just a single line array, which is the same setup as a linear wavefield system. Three slightly overlapping modal regions are used to create an elongated zone parallel to the array. As the angle of wave direction moves closer to the direction of the array, reproduction quality reduces, and an inward moving wave is completely impossible. This problem does not occur for the box enclosure. However a possible disadvantage of the box when reconstructing transient wavefronts is pre-echo artifacts. These could be greatly reduced by constructing a box out of independent line arrays, as found in wavefield systems, and using the walls in pairs according to the planewave being reconstructed. DMC could be useful for designing and controlling line arrays used for large concert events. A similar approach could be used for hemispherical enclosures, constrained by a foam like pack of variously size modal regions, see Fig 8. If listeners are only present near the plane, as in a concert dome, then constraints only need placing in that area, which will improve reconstruction. In the examples so far all the modal regions have consistently represented the same simple soundfield. It is also possible to create modal regions that are separated and each represent a different target field. Fig. 9 shows an example with 4 such regions, 3 constrained to planewaves in different directions, and the other to silence. It is surprising how versatile the soundfield is in being able to configure itself to closely match the competing constraints, without excessive source energy. Of course the reproduction is a well defined soundfield, but it is an unusual one. The regions are not truely independent, they just appear so locally.

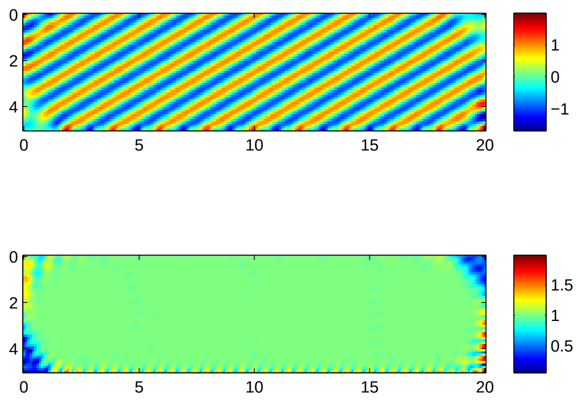

Figure 5: Reconstruction of planewave from the bottom right.

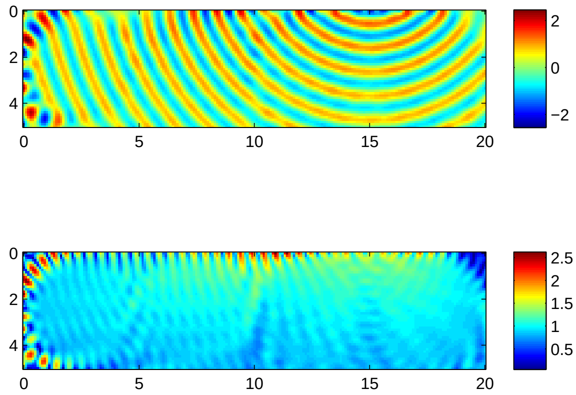

Figure 6: Reconstruction of an exterior point source.

For the listeners in the modal regions, each can experience a different audio environment. If the regions are consistently constrained by the same field then potentially this provides a way to surround several listeners without the full source count required to cover the whole space. The modal regions could be made to track the movement of the listeners, as was proposed for an Ambisonic decoder in [10].

Another intriguing possibility is that a point source can be constrained at several surrounding regions, such that several opposing pairs of listeners all agree the source is at their centre. This is not possible with conventional Ambisonics or wavefield reproduction.

3.5. Encoding

One of the advantages of Ambisonics is that the soundfield is encoded into a discrete number of channels that can be used on different loudspeaker arrays. The same flexibilty can be achieved

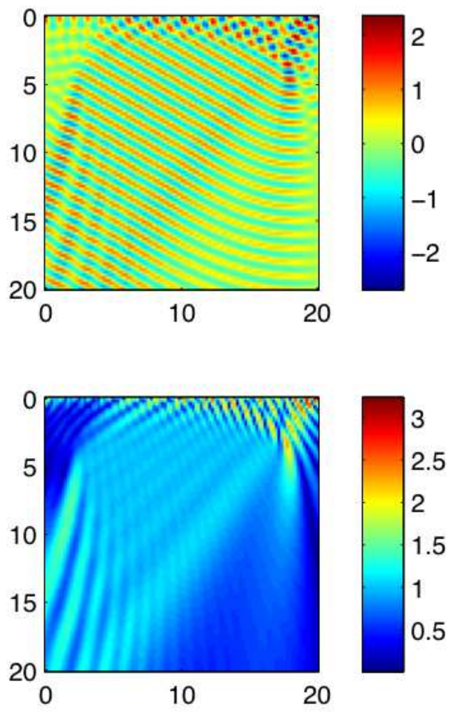

Figure 7: Reconstruction of a planewave from a line array above.

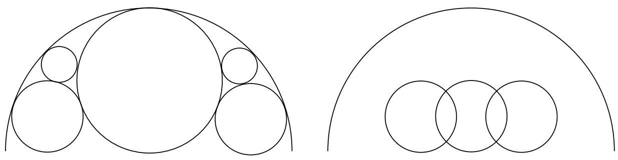

Figure 8: Dome with packed constraints, and ground level only constraints.

with DMC, even though there are multiple constraint centres. First choose the interior point that minimizes the radius required to enclose the reconstruction region, see Fig 10. This point does not have to coincide with any modal constraint centre, and the reconstruction region may be smaller than the interior. The order required to encode the interior is $N = kr$, with r the radius of the enclosing space, [7]. For each mode up to this order, a filter h_{nj} can be calculated for each loudspeaker source s_j. Given an Ambisonic encoded signal B_n, the jth speaker feed is $\sum h_{nj} B_n$. A potential problem with this approach is that it can be wasteful if much of the enclosing region lies outside the interior. A more serious problem arises if an array cannot produce all the possible soundfields accurately in the reconstruction region, as with the line array example. This means there is a set of planewaves that cannot be reproduced. This problem has been discussed previously, [11], a solution is presented here in the current context.

There are two things to consider, modified decoder design, and the treatment of signals that cannot be realized. Taken individually, each mode about the encoding centre will not be real-

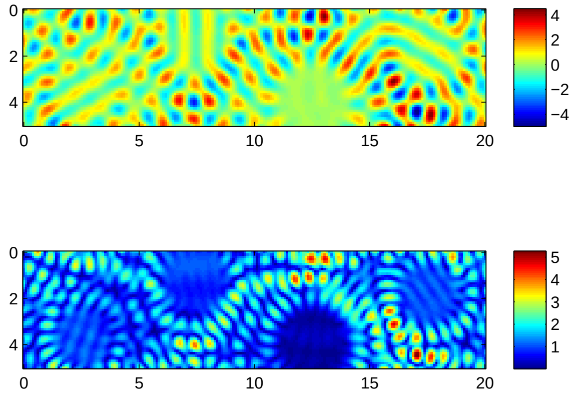

Figure 9: Reconstruction of four independent regions in a rectangular array.

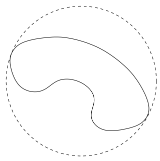

Figure 10: Smallest bounding sphere for a reconstruction region.

izable as there will be non-zero components in prohibited directions. For the inversion process to succeed, a restricted version of each mode is formed $\bar{R}_n(\boldsymbol{r})$, equivalent to $R_n(\boldsymbol{r})$ but zero in the prohibited directions, Fig. 11. In practice the pass window needs smoothing at the edges to reduce aliasing. n order to define the FBE at each constraint, it is necessary to represent $\bar{R}_n(\boldsymbol{r})$ itself as an FBE $\sum \alpha_{nm} R_m(\boldsymbol{r})$. The resulting decoder, with filters $\bar{h}_{nj}$ replacing h_{nj}, will correctly decode any signal with no prohibited directional components, since in that case $\sum B_n R_n(\boldsymbol{r}) = \sum B_n \bar{R}_n(\boldsymbol{r})$. If the decoder is given a signal with prohibited directional components, it is likely to produce badly distorted output. This is avoided by prefiltering the input signal to remove these components. If the input signal is B_n then the intended soundfield is $\sum B_n R_n(\boldsymbol{r})$. Replacing each mode with the restricted version $\bar{R}_n(\boldsymbol{r})$, and re-expanding, leads to a filtered signal $\bar{B}_n = \sum \alpha_{nm} B_n$.

If filtering does remove material, then usually this won't disrupt the perception of the soundfield grossly, unless singular features such as point sources are being encoded, which are in any case not very practical. The focused source, which uses only half the direction space, is practical.

3.6. Interactive design

DMC lends itself to an interactive design process. Ideally the user would be able to place and move around modal regions in an application window and quickly see the resulting soundfield, allowing the appropriate design tradeoffs to be made. Preference could be specified on where source energy should be concentrated and the relative importance of different modal regions.

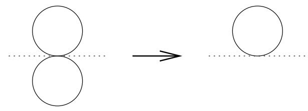

Figure 11: Example of directional response of a mode R_n and restricted version $\bar{R}_n$.

These controls could be implemented by extending the solution procedure.

4. SUMMARY

A review of the properties of various soundfield reproduction methods has led to a hybrid approach, combining simultaneous modal constraints. This is useful for reproducing over any interior shape, and also for controlling the soundfield at localized areas. In the latter case the areas can consistently match one interior target field, or else entirely distinct fields. Distributed Modal Constraints may prove useful for the control of other wave phenomena other than sound. Further study will characterize the capabilities of distributed modal constraints in more detail, including more complex boundaries and interior sources.

5. REFERENCES

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